Exam. Code : 103204
Subject Code : 1109

B.A./B.Sc. 4th Semester

MATHEMATICS

Paper—I (Statics and Vector Calculus)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Do any **FIVE** questions, selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Two forces P and Q acting at a point have a resultant R. If P is doubled, R is doubled and if Q is doubled and reversed in direction, even then R is doubled. Show that $P : Q : R = \sqrt{6} : \sqrt{2} : \sqrt{5}$.

   (b) A given force F is resolved into two components inclined at 45° and α. If the latter component is $\frac{\sqrt{2}}{\sqrt{3}}F$, find α and the other component.

2. (a) A light string of length l is fastened to two points A and B at the same level at a distance a apart. A ring of weight W can slide on the string and horizontal force P is applied to it such that it is in equilibrium vertically below B. Show that $P = \frac{aW}{\ell}$ and tension of the string is $\frac{W(\ell^2 + a^2)}{2\ell^2}$.

(b) If the position of the resultant of two like parallel forces remains unaltered when their positions are interchanged, show that the forces are equal.

3. (a) P and Q are two like parallel forces. If two equal and opposite forces S along any two parallel lines at a distance b apart in the plane of P and Q are combined with them, show that the resultant is displaced through a distance $\frac{bS}{P+Q}$.

(b) A uniform beam 12 metres long and weighing 72 Kg. is supported on two pegs, 1 m and 2 m, respectively from the ends. Where must a weight of 24 Kg. be hung to make the pressure on each peg equal.

4. (a) Forces of magnitude 3, 4, 2 and 1 units act respectively along the sides AB, BC, CD and DA of a square ABCD of sides 3 units. Reduce the system to a force at A and a couple.

(b) A uniform beam of length 2a, rests against a smooth vertical plane over a smooth peg at a distance b from the plane. If θ be the inclination of the beam to the vertical, show that $\sin^3\theta = \frac{b}{a}$.

5. (a) A light ladder rests in limiting equilibrium with its lower-end on a rough floor and the upper-end against a smooth vertical wall. Find how high a man of weight W can climb without slipping takes place.

(b) Find C.G. of a solid right circular cone.

SECTION—B

6. (a) If $\phi = 3x^2y$ and $\psi = xz^2 - 2y$, find grad(grad $\phi \cdot$ grad ψ).

(b) If $u = x + y + z$, $v = x^2 + y^2 = z^2$, $w = xy + yz + zx$, prove that (grad u) $\cdot$ (grad v) $\times$ (grad w) = 0.

7. (a) Find the directional derivative of the function $\phi = x^2 + y^2 + 2z^2$ at the point P(1, –1, 2) in the direction of the line PL, where L is the point (1, 2, 3). Also, find maximum value of directional derivative at P(1, –1, 2).

(b) For the function $f = \frac{y}{x^2 + y^2}$, find the directional derivative making an angle 30° with the positive x-axis at point (0, 1).

8. (a) Find the constants a, b, c so that the vector :

$$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is irrotational.

(b) Given the vector field $\vec{V} = (x^2 - y^2 + 2xy)\hat{i} + (xz - xy + yz)\hat{j} + (z^2 + x^2)\hat{k}$, find curl $\vec{V}$. Show that the vector given by curl $\vec{V}$ at P(1, 2, –3) and Q(2, 3, 12) are orthogonal.

9. (a) State and prove Gauss's divergence theorem.

(b) By transforming to a triple integral evaluate $I = \iint_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$, where S is the closed surface bounded by the planes z = 0, z = b and the cylinder $x^2 + y^2 = a^2$.

10. (a) Verify Green's theorem in the plane for $\oint_C [(3x^2 - 8y^2)\,dx + (4y - 6xy)dy]$, where C is the boundary of the region bounded by $y = \sqrt{x}$ and $y = x^2$.

(b) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS$, where $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ and S is the surface given by $x^2 + y^2 + z^2 = 1$, $z \geq 0$.